DESIGN OF X-BAR CHART FOR BURR DISTRIBUTION UNDER THE REPETITIVE SAMPLING

Muhammad Azam[1], Liaquat Ahmad[1] and Muhammad Aslam[2]

[1]Department of Statistics and Computer Science, University of Veterinary & Animal Sciences, Lahore 54000, Pakistan

[2]Department of Statistics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia

Corresponding Author: aslam_ravian@hotmail.com

***ABSTRACT**: In practice, it is not necessary that the data always follows the normal distribution. In the literature, usual (single) control charts are available for normally and non-normally distributed data. In this manuscript, X-bar control chart under the repetitive sampling scheme is proposed for Burr type XII distribution. The average run length and the co-efficient of the proposed plan are determined for various values of sample size and specified average run length. The performance of the proposed plan is compared with the usual X-bar control chart for Burr type XII distribution.*

Key words: Single control chart; repetitive sampling; average run length; Burr type XII distribution

INTRODUCTION

The statistical techniques have been widely used in the industries of the world for the betterment of the product. These are used to monitor the quality of the product from the raw material to the final stage. Among these techniques, control charts and acceptance sampling plan have been widely used in the industries. First one is used to monitor the quality of the product during the manufacturing process. This leads to the industrial engineers to enhance the quality of the product. Second one has been used for the inspection of the final product. For the high quality assurance, both statistical techniques are very necessary for the industries of the world. It may not possible to produce the product according to the given specifications limits without the control charts. Among these control charts, usual X-Bar control chart proposed by Dr. W. A. Shewhart in 1924 has been widely used in process monitoring due to its simplicity. More details about the usual control chart can be seen in [1,2,3,4,5,6,7,8]

The two important sampling schemes; single sampling and double sampling have been wieldy used in the control chart. Although, the single sampling scheme is widely used in the industries due it is simple applications but the double sampling is more efficient than the single sampling. The double sampling provides the less value of average run length and average sample number as compared to single sampling. So, the control charts proposed under the double sampling are more efficient than indicate out-of-control signal quickly than the usual control chart. [9] proposed the double sampling control chart for attributes. According to [10] usual Shewhart X-bar control charts are more efficient in detecting the shift for large sample size. However, the increase in sample size increases the cost of the inspection.

The idea of the repetitive group sampling (RGS) plan is originally given by[11]. The operation of the RGS plan is similar to the sequential sampling plan. The RGS plan simple in application. Moreover, the RGS plan is more efficient than single and double sampling plan. [12] proposed the RGS plan variable inspection when the quality of characteristics follow the normal distribution with known and unknown standard deviation. They showed that the variable RGS plan is more efficient than variable single sampling plan and attribute RGS plan. Further, they also compared the variable RGS plan with variable single sampling plan, variables double sampling plan and variable sequential sampling plan. The variable RGS is better than variable single sampling plan and variable double sampling plan [12]. But, the variable sequential sampling plan is more efficient than the variable RGS plan.

The idea of the repetitive should not be mixed with the double and variable sampling internal (VSI). As in double sampling second sample is taken if the experimenters cannot reach on decision in first sample. While in repetitive sampling plan, the procedure is repeated if the experiment cannot make the decision. On the other hand, the repetitive sampling is also different in operation than the VSI sampling. According to [13] "The idea of varying the sampling interval (VSI) for an X-bar chart is intuitive. If a sample point falls in the warning region it is reasonable to wait less time to take the nest sample because the process can be demanding. On the other hand, if the sample point falls in the central region it is reasonable to wait more time to take the next sample because there is no evidence that the process needs adjustment". Furthermore, the control charts using the double sampling and VSI are more complicated for administrative point of view. On the other hand, the application of the repetitive sampling in the area of control chart is easy as compared to the double and VSI sampling. Therefore, the use of repetitive sampling in the control chart theory may minimize the cost and time than the single, double, VSI and sequential sampling schemes. So, the proposed control charts using the repetitive sampling scheme will be different from the control charts using the double sampling and VSI in the literature. Recently, [14] designed the t-chart using the repetitive sampling and showed the superiority of proposed plan over the existing control chart. [15] presented the complete structure of X-bar control chart for the process capability index under the repetitive sampling.

By exploring the literature, we can found that the control charts are available only for single and double sampling plans for normal or the non-normal data [16]. No attempt has been made to introduce the X-bar control chart using RGS plan in the area of control chart. In this paper, we will use the idea of RGS plan to propose a new control chart using the Burr type XII distribution. We will compare the efficiency of the proposed control chart with the existing sampling for the Burr type XII distribution. The rest of the paper is organized as follows: proposed plan for the non-normal distribution is proposed in Section 2. Comparative study is given in Section 3. The concluding remarks are given in the last section.

2. DESIGN OF CONTROL CHART UNDER THE RGS PLAN

In this section, we will propose a new X bar control chart when the quality characteristic follows the non-normal distribution using the RGS plan.

Step 1: Take a sample of size n. Calculate the sample mean $\bar{X}$.

Step 2: Declare out-of-control if $\bar{X} \geq UCL_1$ or $\bar{X} \leq LCL_1$ (UCL_1 and LCL_1 are called outer control limits). Declare in-control if $LCL_2 \leq \bar{X} \leq UCL_2$ (UCL_2 and LCL_2 are called inner control limits). Otherwise, go to Step 1 and repeat the process.

The proposed control chart is the extension usual X-bar control chart. The proposed control chart reduces to the traditional X-bar control chart when $UCL_1 = LCL_1$ and $L_2 = LCL_2$.

The cumulative distribution function (cdf) of the Burr distribution is given as [17]

$$F(y) = 1 - \frac{1}{(1+y^c)^q}, \qquad y \geq 0 \tag{1}$$

where c and q present the skewness and kurtosis of Burr distribution.

First we redefine the control limits that are taken from [16] under the proposed control charts, and given as follows

$$UCL_1 = \mu_0 + k_1 \frac{\sigma}{\sqrt{n}} \tag{2}$$

$$LCL_1 = \mu_0 - k_1 \frac{\sigma}{\sqrt{n}} \tag{3}$$

$$UCL_2 = \mu_0 + k_2 \frac{\sigma}{\sqrt{n}} \tag{4}$$

$$LCL_2 = \mu_0 - k_2 \frac{\sigma}{\sqrt{n}} \tag{5}$$

where μ_0 is the process average, σ is the process standard deviation, k_1 and k_2 are the co-efficient of the proposed control chart.

From [16] , we have the relation of $\bar{X}$ as

$$\bar{X} = \mu_0 + (Y - M)\frac{\sigma}{S\sqrt{n}} \tag{6}$$

where, $\bar{X}$ is the sample mean, S is the standard deviation of Burr distribution, Y is the random variable of Burr distribution and M is the mean of Burr distribution.

The probability that the process is declared to be in control is given by

$$P_{in}(p) = \frac{P(LCL_2 < \bar{X} < UCL_2)}{1 - P_{rep}(p)} \tag{7}$$

where $P_{rep}(p)$ is the probability of the repetition and is given as follows

$$P_{rep}(p) = P\{LCL_1 \leq \bar{X} \leq LCL_2\} + P\{UCL_2 \leq \bar{X} \leq UCL_1\} \tag{8}$$

and

$P(LCL_2 < \bar{X} < UCL_2)$ is the probability of in-control for the traditional control chart.

Based on the above information, we want to derive the probability that the process is in control under the proposed control chart scheme. The probability that the process is announced to be in control for first sample can be calculated as follows

$$P(LCL_2 \leq \bar{X} \leq UCL_2) = P\left(\mu_0 - k_2\frac{\sigma}{\sqrt{n}} \leq \mu_0 + (Y - M)\frac{\sigma}{S\sqrt{n}} \leq \mu_0 + k_2\frac{\sigma}{\sqrt{n}}\right) \tag{9}$$

After some simplification, we have

$$P(LCL_2 \leq \bar{X} \leq UCL_2) = P(M - k_2 S \leq Y \leq M + k_2 S) \tag{10}$$

Finally, using the cdf of Burr distribution given in Eq. (1), the P_{in} can be rewritten as

$$P(LCL_2 \leq \bar{X} \leq UCL_2) = P\left(\frac{1}{[1+(M-k_2S)^c]^q} - \frac{1}{[1+(M+k_2S)^c]^q}\right) \tag{11}$$

After some simplification, $P_{rep}(p)$ can be written as follows

$$P_{rep}(p) = P\left(\frac{1}{[1+(M-k_1S)^c]^q} - \frac{1}{[1+(M-k_2S)^c]^q}\right) + P\left(\frac{1}{[1+(M+k_2S)^c]^q} - \frac{1}{[1+(M+k_1S)^c]^q}\right) \tag{12}$$

Finally, the probability that the process is decaled in control under the proposed control chart is given as follows

$$P_{in}(p) = \frac{\left(\frac{1}{[1+(M-k_2S)^c]^q} - \frac{1}{[1+(M+k_2S)^c]^q}\right)}{1 - \left[\left(\frac{1}{[1+(M-k_1S)^c]^q} - \frac{1}{[1+(M-k_2S)^c]^q}\right) + \left(\frac{1}{[1+(M+k_2S)^c]^q} - \frac{1}{[1+(M+k_1S)^c]^q}\right)\right]} \tag{13}$$

when the process mean has shifted to $\mu = \mu_0 + C\sigma$, then the probability that process is in control using the first sample is given as

$$P(LCL_2 \leq \bar{X} \leq UCL_2/\mu = \mu_0 + C\sigma) = P\left(\frac{1}{\left[1+\left(M-(k_2S+CS\sqrt{n})\right)^c\right]^q} - \frac{1}{\left[1+\left(M+(k_2S-CS\sqrt{n})\right)^c\right]^q}\right) \tag{14}$$

and

$$P_{rep}(p) = P\left(\frac{1}{\left[1+\left(M-(k_1S+CS\sqrt{n})\right)^c\right]^q} - \frac{1}{\left[1+\left(M-(k_2S+CS\sqrt{n})\right)^c\right]^q}\right) + P\left(\frac{1}{\left[1+\left(M+(k_2S-CS\sqrt{n})\right)^c\right]^q} - \frac{1}{\left[1+\left(M+(k_1S-CS\sqrt{n})\right)^c\right]^q}\right) \tag{15}$$

Finally, the probability the process is in control (P_{in}^*) for the shifted mean is given as follows

$$P_{in}^* = \frac{\left(\frac{1}{\left[1+\left(M-(k_2S+CS\sqrt{n})\right)^c\right]^q} - \frac{1}{\left[1+\left(M+(k_2S-CS\sqrt{n})\right)^c\right]^q}\right)}{1 - \left[\left(\frac{1}{\left[1+\left(M-(k_1S+CS\sqrt{n})\right)^c\right]^q} - \frac{1}{\left[1+\left(M-(k_2S+CS\sqrt{n})\right)^c\right]^q}\right) + \left(\frac{1}{\left[1+\left(M+(k_2S-CS\sqrt{n})\right)^c\right]^q} - \frac{1}{\left[1+\left(M+(k_1S-CS\sqrt{n})\right)^c\right]^q}\right)\right]} \tag{16}$$

The average sample size (ASS) of the proposed control chart is given as below

$$ASS(p) = \frac{n}{1 - \left[P\left(\frac{1}{\left[1+(M-k_1S\sqrt{n})^c\right]^q} - \frac{1}{\left[1+(M-k_2S\sqrt{n})^c\right]^q}\right) + P\left(\frac{1}{\left[1+(M+k_2S\sqrt{n})^c\right]^q} - \frac{1}{\left[1+(M+k_1S\sqrt{n})^c\right]^q}\right)\right]} \tag{17}$$

The average run length (ARL) when the process is in control called ARL_0 for the proposed control chart is given as follows

$$ARL_0 = \frac{1}{1 - \left\{\frac{\left(\frac{1}{[1+(M-k_2S)^c]^q} - \frac{1}{[1+(M+k_2S)^c]^q}\right)}{1 - \left[\left(\frac{1}{[1+(M-k_1S)^c]^q} - \frac{1}{[1+(M-k_2S)^c]^q}\right) + \left(\frac{1}{[1+(M+k_2S)^c]^q} - \frac{1}{[1+(M+k_1S)^c]^q}\right)\right]}\right\}} \tag{18}$$

The ARL when the process is out of control called ARL_1 for the proposed control chart is given as follows

$$ARL_1 = \frac{1}{1 - \left\{\frac{\left(\frac{1}{\left[1+\left(M-(k_2S+CS\sqrt{n})\right)^c\right]^q} - \frac{1}{\left[1+\left(M+(k_2S-CS\sqrt{n})\right)^c\right]^q}\right)}{1 - \left[\left(\frac{1}{\left[1+\left(M-(k_1S+CS\sqrt{n})\right)^c\right]^q} - \frac{1}{\left[1+\left(M-(k_2S-CS\sqrt{n})\right)^c\right]^q}\right) + \left(\frac{1}{\left[1+\left(M+(k_2S-CS\sqrt{n})\right)^c\right]^q} - \frac{1}{\left[1+\left(M+(k_1S-CS\sqrt{n})\right)^c\right]^q}\right)\right]}\right\}} \tag{19}$$

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Table 1: Average Run Lengths of Proposed control charts when $n = 10, 20$ and $r_0 = 100, 200, 300$

Shift (f)	ARL_0 = 100, k_1 = 2.8132, k_2 = 0.6271 ASSo=21.12		ARL_0 = 200, k_1 = 3.9151, k_2 = 0.2997 ASSo=42.29		ARL_0 = 300, k_1 = 3.9524, k_2 = 0.5480 ASSo=23.97	
	ASS_1	ARL_1	ASS_1	ARL_1	ASS_1	ARL_1
0.0	21.12	100.03	42.29	200.04	23.97	300.03
0.1	21.62	51.23	43.10	43.68	24.52	67.45
0.2	23.80	23.26	46.56	13.11	26.81	20.59
0.3	27.66	10.34	50.66	4.99	30.36	7.68
0.4	32.45	4.69	51.30	2.40	33.64	3.44
0.5	35.45	2.38	45.24	1.50	33.86	1.91
1.0	11.11	1.00	11.00	1.00	10.96	1.00
1.5	5.87	1.00	5.80	1.00	5.99	1.00
2.0	6.52	1.00	5.66	1.00	6.29	1.00
2.5	5.81	1.00	5.27	1.00	5.52	1.00
3.0	6.89	1.00	5.43	1.00	5.42	1.00
Shift (f)	ARL_0 = 100, k_1 = 2.9731, k_2 = 0.3915 ASSo=65.11		ARL_0 = 200, k_1 = 3.0658, k_2 = 0.6479 ASSo=41.27		ARL_0 = 300, k_1 = 3.9689, k_2 = 0.5954 ASSo=44.52	
	ASS_1	ARL_1	ASS_1	ARL_1	ASS_1	ARL_1
0.0	65.11	100.00	41.27	200.00	44.52	300.07
0.1	69.08	34.25	43.88	66.45	46.89	41.04
0.2	83.94	10.34	53.90	18.94	55.11	9.34
0.3	101.48	3.23	70.12	5.19	63.76	3.04
0.4	92.61	1.47	76.42	1.90	60.37	1.51
0.5	60.10	1.09	57.46	1.18	45.27	1.12
1.0	11.75	1.00	11.89	1.00	12.57	1.00
1.5	11.79	1.00	13.16	1.00	12.86	1.00
2.0	11.71	1.00	11.65	1.00	10.46	1.00
2.5	18.91	1.00	18.72	1.00	15.77	1.00
3.0	19.99	1.00	19.99	1.00	19.92	1.00

Table 2: Average Run Lengths of Proposed control charts when $n = 30, 40$ and $r_0 = 100, 200, 300$

Shift (f)	ARL_0 = 100, k_1 = 2.7638, k_2 = 0.7374 ASSo=55.17		ARL_0 = 200, k_1 = 2.9620, k_2 = 0.8970 ASSo=47.42		ARL_0 = 300, k_1 = 3.1334, k_2 = 0.8662 ASSo=48.79	
	ASS_1	ARL_1	ASS_1	ARL_1	ASS_1	ARL_1
0.0	55.17	100.00	47.42	200.00	48.79	300.02
0.1	59.92	29.90	51.55	53.55	53.34	74.90
0.2	77.01	7.58	67.10	12.22	70.48	14.68
0.3	93.38	2.29	87.46	3.04	93.75	3.21
0.4	76.04	1.22	78.00	1.34	82.95	1.34
0.5	47.20	1.03	48.57	1.05	49.88	1.05
1.0	18.55	1.00	19.21	1.00	18.94	1.00
1.5	17.66	1.00	17.78	1.00	17.41	1.00
2.0	28.33	1.00	27.67	1.00	26.97	1.00
2.5	30.00	1.00	29.99	1.00	29.99	1.00
3.0	30.00	1.00	30.00	1.00	30.00	1.00
Shift (f)	ARL_0 = 100, k_1 = 2.7388, k_2 = 0.8043 ASSo=65.51		ARL_0 = 200, k_1 = 2.9679, k_2 = 0.8790 ASSo=64.22		ARL_0 = 300, k_1 = 3.9929, k_2 = 0.6744 ASSo=79.86	
	ASS_1	ARL_1	ASS_1	ARL_1	ASS_1	ARL_1
0.0	68.51	100.00	64.22	200.01	79.86	300.04
0.1	76.42	24.75	72.10	42.78	88.92	22.27
0.2	102.91	5.25	100.59	7.61	112.97	3.83
0.3	112.85	1.63	119.80	1.87	107.64	1.41
0.4	74.46	1.08	79.24	1.10	69.57	1.05
0.5	44.17	1.00	44.59	1.01	43.69	1.00
1.0	28.06	1.00	28.82	1.00	26.54	0.76
1.5	28.27	1.00	26.63	1.00	21.62	0.99
2.0	39.95	1.00	39.91	1.00	39.07	1.00
2.5	40.00	1.00	40.00	1.00	40.00	1.00
3.0	68.51	1.00	40.00	1.00	40.00	1.00

Table 3: Comparisons of ARLs between the proposed chart and the usual chart.

Shift (f)	ARL_0 = 200, n = 10		ARL_0 = 200, n=20	
	Existing Chart k = 2.7936	Proposed Chart k_1 = 3.9151, k_2 = 0.2997	Existing Chart k = 2.7937	Proposed Chart k_1 = 3.0658, k_2 = 0.6479
0.0	200.01	200.04	200.04	200.00
0.1	105.95	43.68	79.29	66.45
0.2	53.10	13.11	30.76	18.94
0.3	27.54	4.99	12.75	5.19
0.4	14.75	2.40	5.79	1.90
0.5	8.24	1.50	2.96	1.18
1.0	1.11	1.00	1.00	0.96
1.5	1.00	0.97	1.00	0.76
2.0	1.00	0.89	1.00	0.97
2.5	1.00	0.95	1.00	1.00
3.0	1.00	1.00	1.00	1.00

Table 4: Comparisons of ARLs between the proposed chart and the usual chart.

Shift (f)	ARL_0 = 300, n = 10		ARL_0 = 300, n=20	
	Existing Chart k = 2.9421	Proposed Chart k_1 = 3.9524, k_2 = 0.5480	Existing Chart k = 2.9421	Proposed Chart k_1 = 3.9689, k_2 = 0.5954
0.0	300.02	300.03	300.03	300.07
0.1	148.48	67.45	110.30	41.04
0.2	72.97	20.59	40.97	9.34
0.3	36.39	7.68	15.76	3.04
0.4	18.48	3.44	6.66	1.51
0.5	9.79	1.91	3.22	1.12
1.0	1.12	1.00	1.00	1.00
1.5	1.00	1.00	1.00	1.00
2.0	1.00	1.00	1.00	1.00
2.5	1.00	1.00	1.00	1.00
3.0	1.00	1.00	1.00	1.00

The average sample number (ASS) under the proposed control chart is given as follows

$$ASS(p) = \frac{n}{1-\left[P\left(\frac{1}{\left[1+\left(M-k_1 S\sqrt{n}\right)^c\right]^q}-\frac{1}{\left[1+\left(M-k_2 S\sqrt{n}\right)^c\right]^q}\right)+P\left(\frac{1}{\left[1+\left(M+k_2 S\sqrt{n}\right)^c\right]^q}-\frac{1}{\left[1+\left(M+k_1 S\sqrt{n}\right)^c\right]^q}\right)\right]} \quad (20)$$

We determined ARL_1, ASS and both co-efficient for various values of sample size $(n = 10,20)$, specified ARL $(r_0 = 100,200,300)$ and for various Shifts in Table 1. In Table 2, the same parameters are provided for $n = 75$ and 100. For the construction of Tables 1 and 2, we considered the specified values of $M = 0.5951$, $S = 0.1801$, $c = 4$ and $q = 6$ from [17]. We used the simulation procedure to find the control chart parameters in R. The control chart parameters can be determined for any values of $c = 4$ and $q = 6$. The read may request the authors for the R program. We used following algorithm to find the control chart parameters.

Step-1: specify n and r_0.

Step-2: Find the values of k_1 and k_2 $(k_1 > k_2)$ for which ARL_0 using Eq. (18) is close to r_0.

Step-3: Using the selected values of k_1 and k_2 in step-2, find ARL_1 for various values of shifts.

Tables 1-2 are around here

From Tables 1-2, we can see that for same n as C moves from 0 to 3.0, there is decreasing trend in the values of ARL_1. Also note that k_1 is larger than 3 and k_2 is less than one. When sample size increases from 10 to 40, we note the much reduction in ARL_1.

3. COMPARATIVE STUDY

The advantage of the proposed control chart is discussed with the traditional X-bar control chart for the Burr type XII distribution. We determined ARL_1 values for the two control chart using the same values of specified control chart parameters and presented in Tables 3-4.

Tables 3-4 are around here

From Tables 3-4, we can note that the proposed control chart provides the less values of ARL_1 as compared to the traditional single control chart for the same values of specified parameters. It is true for all values of sample size. For example, when n =20, r_0=300 and C=0.1, the ARL_1 from the proposed plan is 41 and from the existing control chart is 110. So, the proposed control chart is more efficient than the traditional control chart and indicates out of control single almost 7 times quicker than the traditional control chart.

4. CONCLUDING REMARKS

A new control chart using the RGS plan is proposed in the paper. The necessary measurements of the proposed plan are derived. The tables are presented for various values of sample size and specified ARLs for practical use. The trend in ARLs values if discussed. The advantage of the proposed plan over the traditional control chart is discussed. The proposed control chart provides the smaller values of ARLs as compared to the traditional X-bar control chart for Burr type XII distribution. The proposed control chart is quite effective in ARLs. The proposed control chart is quicker than the usual control chart in indicating out-of-control single. The use of the control chart in industries can help in reducing the non-conforming items. Therefore, it is strongly recommended to the industrial engineering to use the proposed plan for monitoring the process of manufacturing. The proposed control chart is new one therefore its measures can be deriving for various control charts as a future research. The authors are working on the extension of the proposed control chart when the data is correlated. The use of other distribution such as Weibull distribution and gamma distribution is recommended for further research.

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